Exploring the Dimensions of Active Management

The covariance structure of active return.

Dorsey D. Farr

In 2003 Waring and Siegel provided a provocative discussion of the problem of multi-manager portfolio construction. They note that anyone constructing a multi-manager portfolio is either implicitly or explicitly solving an optimization problem like:

$$\max_{\pi_i} \sum_{i=1}^{n} \pi_i \alpha_i - \lambda \Omega[\pi(\pi_1, \pi_2, \ldots, \pi_n)]$$

where π_i is the allocation to manager i, α_i is the average active return for manager i, λ is a parameter governing the investor's willingness to accept tracking error risk, and Ω is a measure of the aggregate active risk, which is a function of the active risk for each manager and the covariance structure of active return for all the managers.

While it is not uncommon to consider portfolio construction as an optimization problem, few investors actually carry out an explicit optimization analysis. Waring and Seigel (WS) should be commended for encouraging investors to confront the *implicit* assumptions built into a particular portfolio structure.

WS focus mainly on two dimensions of active management—alpha and tracking error. Indeed, their subtitle claims "alpha and active risk are the only things that matter," despite the fact that the covariance structure of active return—what one might refer to as the third dimension of active management—is theoretically of equal importance.

WS conclude on the basis of the results of their optimization analysis that an optimal portfolio structure (for modest levels of active risk) includes a large allocation to risk-controlled active strategies (i.e., enhanced index strategies) but little role for index funds, traditional active managers, or concentrated portfolios.

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Before investors attempt to implement the portfolio structure suggested by the optimization results in WS, they might want to keep in mind that the results of any optimization are only as good as the inputs (the assumptions). Indeed, the optimization output can magnify any erroneous assumptions in the inputs. Garbage in, garbage out.

I examine the sensitivity of an optimal portfolio structure to various assumptions about the covariance structure of active return. I show that the WS results could be misleading if there is either a sufficient degree of diversification in the sources of active return or a sufficient number of unique sources of active return. Sensitivity analysis reveals that the optimal portfolio mix varies substantially with both the degree of diversification in active returns and the number of unique sources of active return, suggesting that the WS result may overstate the case for risk-controlled active strategies.

I first reproduce the WS results, and explore the impact of altering the assumptions about the correlation of active returns. Next I examine the impact of changing the assumptions about the number of unique sources of active return. Finally I study the optimal portfolio structure using a highly stylized example incorporating multiple sources of active return with a more elaborate correlation structure.

DIVERSIFIED ALPHA: COVARIANCE STRUCTURE OF ACTIVE RETURN

Waring and Siegel point out that on average the correlation among the active returns (alphas) of active managers is approximately zero, leading them to assume for the purpose of their optimization analysis that all sources of active return are uncorrelated. Exhibit 1 provides statistics on optimal portfolio structures based on this WS assumption.[1]

For modest levels of active risk (e.g., tracking error in the neighborhood of 2% to 3%), the optimal portfolio combination includes a large allocation to risk-controlled strategies, with a limited role for traditional active and concentrated strategies. WS note that the optimal allocation to a manager increases with the level of active return and declines with the variance of active return. When choosing among managers with equal information ratios and uncorrelated active returns, investors should allocate more capital to the managers with lower levels of active risk.

While the WS example is a reasonable base case, there may be other scenarios worth consideration. Although active returns are by definition uncorrelated with a manager's benchmark, they are not necessarily uncorrelated across managers. Differences in correlation structure among active returns will lead to differences in optimal portfolio structure.

To illustrate this point, I show how the optimal portfolio structure changes when we allow the parameter governing the correlation between the active returns of the value and growth strategies to vary within a range of −1.0 to +1.0, while maintaining the assumption that all other active return correlations are equal to zero.[2]

Exhibits 2 and 3 show how the optimal portfolio mix and the information ratio vary with the degree of diversification in the active returns. As one might expect, the information ratio increases with the level of diversification associated with the sources of active return, while the optimal allocation to risk-controlled and index strategies declines with the degree of diversification.

With independent active returns, there is a large role for enhanced strategies. This is the result highlighted by WS. When the sources of active return are positively correlated, enhanced and indexed strategies play an even greater role. With sufficiently diversified sources of active return, however, the optimal allocation to index managers *and* risk-controlled active strategies declines rapidly with the level of expected active risk. The sensitivity analysis shows that with a moderate level of diversified active return (e.g., a correlation in the neighborhood of −0.25) and a 2.5% active risk budget, the optimal allocation to index strategies and risk-controlled strategies is less than 30%.

One might naturally question whether it is realistic to assume such diversification benefits. To address this

EXHIBIT 1
Optimal Portfolio Allocations (%)

	Expected Active Risk (Tracking Error, bp)				
	100	150	200	250	300
Index	43.9	15.8	0.0	0.0	0.0
RCA	33.3	50.0	52.4	38.4	25.7
Value	10.0	15.0	20.7	26.5	31.7
Growth	10.0	15.0	20.7	26.5	31.7
Concentrated	2.8	4.2	6.2	8.6	10.9
Information Ratio	1.00	1.00	0.99	0.96	0.92

Note: Assumes all sources of active return are uncorrelated. RCA represents risk-controlled active strategies (e.g., enhanced indexing).

EXHIBIT 2

Optimal Allocation to Index Funds and Risk-Controlled Strategies for Various Active Risk Targets

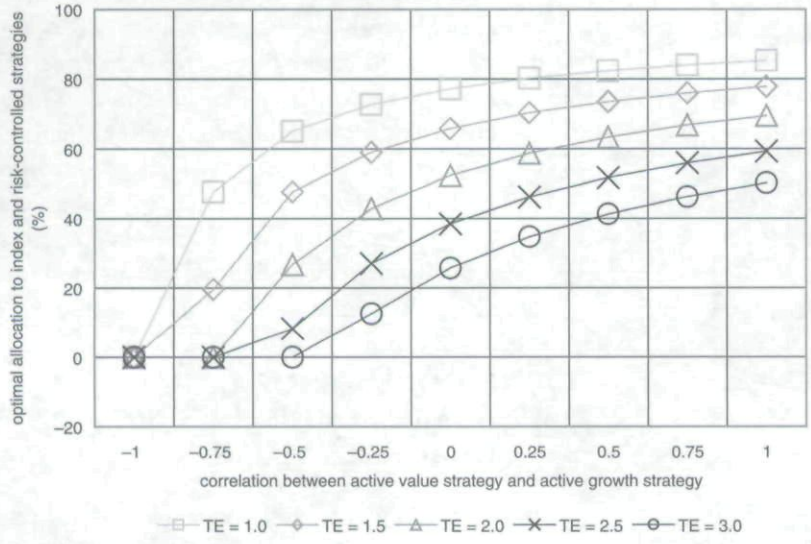
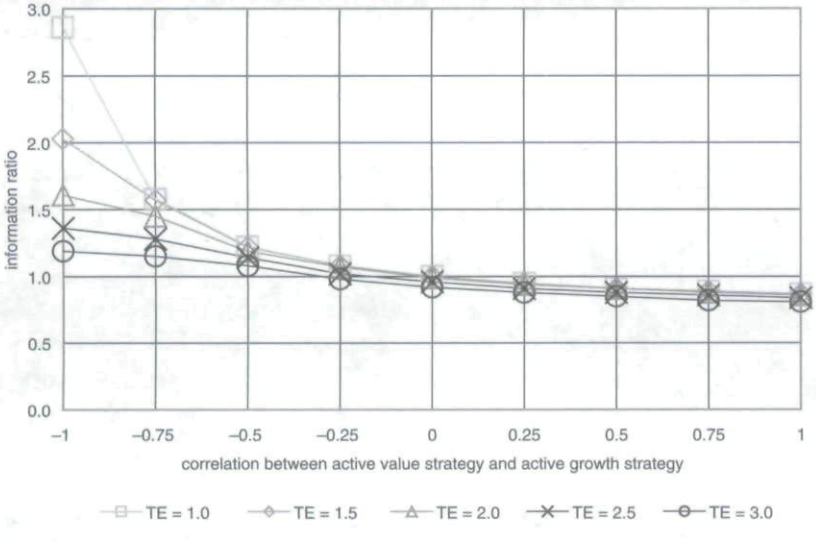


EXHIBIT 3

Information Ratio for Optimal Portfolios with Various Active Risk Targets



question, Exhibit 4 presents the distribution of pairwise correlations among statistically significant, positive mean active returns from a universe of 1,821 domestic equity strategies for the five-year period ended March 31, 2001.[3] The average correlation is very close to zero, but there is clearly a *distribution* of correlations; that is, some are negative, some close to zero, and some positive.

To be sure, a distribution of realized values does not by itself imply a distribution of expected values. Yet the range of observed correlations suggests there are correlations among active returns that are statistically different from zero (both positive and negative).[4]

When we form expectations about future correlations, seeing past correlations among particular strategies that are statistically different from zero (either positive or negative) would generally lead us to expect that such strategies are not independent. This does not suggest that negatively correlated sources of alpha are easy to identify. Active management is indeed a challenging game. Yet as correlations that are statistically different from zero are observed in practice, investors constructing portfolios should understand the implications of alternative correlation structures for optimal portfolio structure (especially to the extent that such correlations might be expected to prevail in the future).

While the active returns for managers may not exhibit any correlation *on average*, part of the art of portfolio construction is selecting good complements. Given the distribution shown in Exhibit 4, it seems perfectly reasonable to expect we could identify managers in a particular asset class with a modest negative correlation among active returns. Because the market tends to exhibit somewhat fickle preferences, it may systematically reward one particular active strategy at exactly the time that it penalizes another strategy, and vice versa. Similarities and differences in the information used by managers and the methodologies managers use to process information create opportunities for non-zero correlations—both positive and negative—among active returns.[5]

Our point here is to emphasize that there are a range of correlations, and an ideal multi-manager portfolio should pair up negatively correlated sources of active returns and avoid combining active return sources that are likely to be positively correlated. I suspect that most manager selectors probably fail in this endeavor, as personal or institutional biases cause them to pick managers of the same breed. This behavior results in positively

EXHIBIT 4

Distribution of Pairwise Correlations of Active Returns

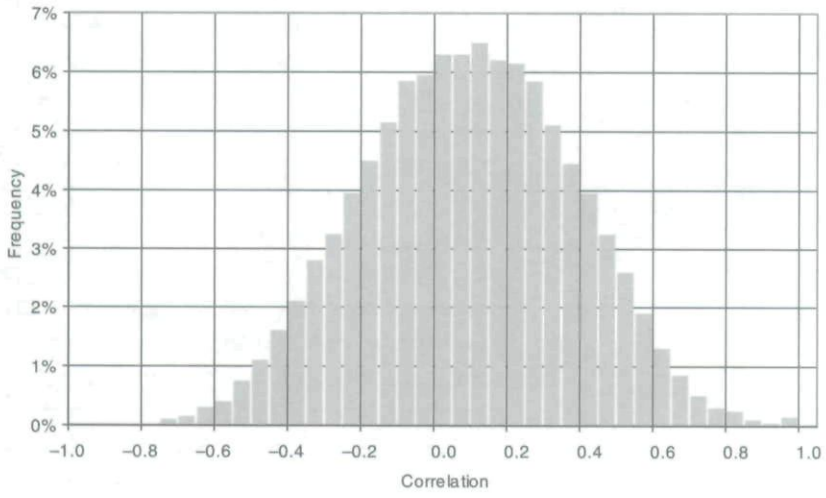
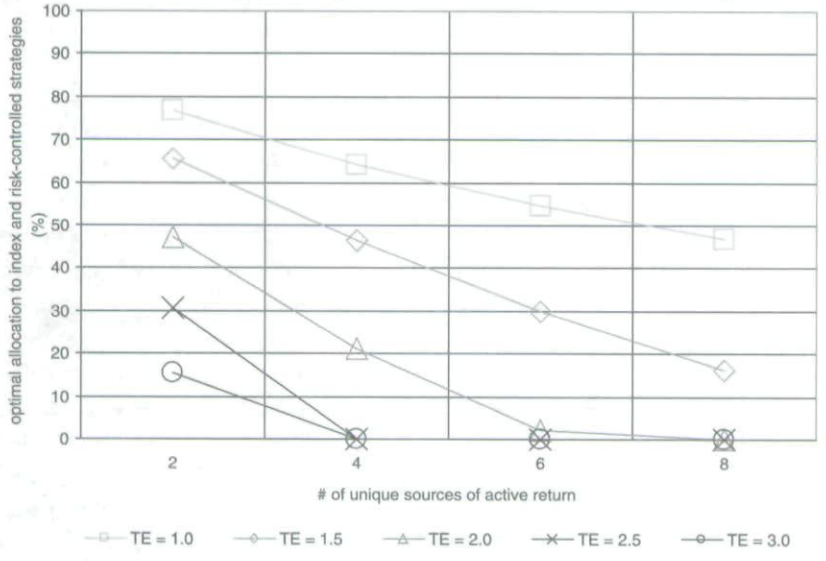


EXHIBIT 5

Optimal Allocation to Indexed and Risk-Controlled Strategies for Various Active Risk Targets



correlated sources of active return and reinforces the WS point that indexes and enhanced indexes should dominate a portfolio.

Much like asset allocation, greater diversification results in more efficient portfolio combinations. As the level of diversification among the sources of active return increases, the active return (alpha)/active risk (tracking error) trade-off becomes more attractive, and the optimal allocation to index strategies and risk-controlled active strategies is reduced. A carefully constructed portfolio of active managers should include active managers who are good complements to one another—those whose active returns are negatively correlated.

To the extent that one can identify such combinations, the role of indexes and risk-controlled strategies is greatly diminished. To the extent that manager selection processes favor managers of a particular type, who are likely to exhibit positively correlated excess returns, the role of index and enhanced strategies increases.

UNIQUENESS OF ALPHA: NUMBER OF DISTINCT SOURCES OF ACTIVE RETURN

Shifting gears, I now explore how multiple (unique) sources of active return might lead to superior portfolio construction. To isolate the importance of the number of unique sources of active return (rather than the degree of uniqueness), we adopt the Waring and Siegel assumption that the active returns of each manager in the portfolio are uncorrelated.

Exhibits 5 and 6 show how the optimal allocation to risk-controlled and index strategies and the information ratio for various levels of tracking error vary with the number of unique sources of active return. The optimal allocation to risk-controlled strategies and index strategies declines with the number of unique sources of active return, while the information ratio increases with it. The latter result is well known in active management—increasing the number of unique bets produces a higher information ratio.

As in the case of negatively correlated sources of active return, identifying a large number of independent sources of active return is likely quite difficult. Some asset classes have relatively few degrees of freedom (implying that a large number of independent alpha sources may be difficult to identify), while others may prove more fruitful. Portfolios of 10 to 12 managers likely entail a lot of overlap, and manager selectors probably fall victim to their own biases.

We should devote more energy to uncovering unique sources of active return, ideally to identifying negatively

EXHIBIT 6
Information Ratio for Various Active Risk Targets

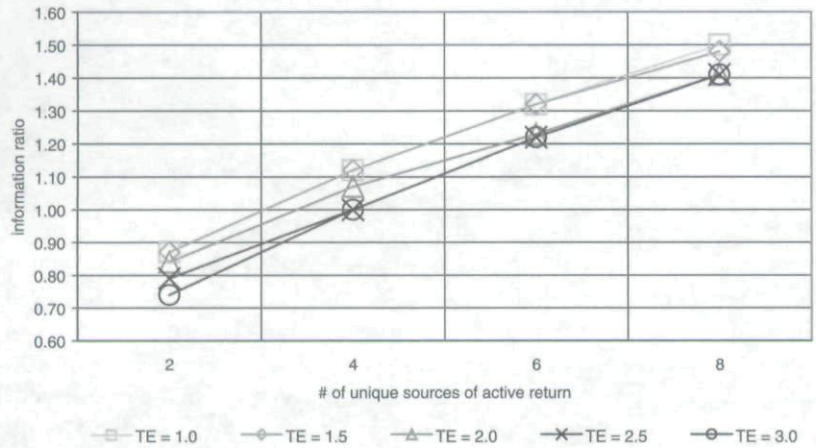


EXHIBIT 7
Covariance Structure of Active Return

	RCA	Value	Growth	Contrarian	Momentum	Concentrated
RCA	1.00	–	–	–	–	–
Value	0.00	1.00	–	–	–	–
Growth	0.00	-0.10	1.00	–	–	–
Contrarian	0.00	0.10	-0.20	1.00	–	–
Momentum	0.00	-0.20	0.10	-0.30	1.00	–
Concentrated	0.00	0.00	0.00	0.00	0.00	1.00

EXHIBIT 8
Optimal Portfolio Allocations (%)

	Expected Active Risk (Tracking Error, bp)				
	100	150	200	250	300
Index	32.6	0.0	0.0	0.0	0.0
RCA	23.9	34.4	9.6	0.0	0.0
Value	9.2	13.9	19.0	20.2	19.3
Growth	9.2	13.9	19.0	20.2	19.3
Contrarian	11.6	17.4	23.7	25.2	24.1
Momentum	11.6	17.4	23.7	25.2	24.1
Concentrated	2.0	3.0	5.1	9.1	13.3
Information Ratio	1.40	1.40	1.33	1.24	1.12

Note: Percentages may not add to 100 because of rounding.

correlated sources of active return. Despite its difficulty, if one is going to expend effort on hiring active managers, it is important to pursue the right goal.

AN EXAMPLE—MULTIPLE DIVERSIFIED SOURCES OF ACTIVE RETURN

I now study the optimal portfolio structure problem using a highly stylized example where there are multiple sources of active return with a more elaborate correlation structure. I assume there are six unique sources of active return, including various aggressive active strategies. I incorporate substyles of value and growth by adding a contrarian manager and a momentum manager, rather than limit the active managers to the broad categorizations of a value strategy and a growth strategy.[6]

I assume the active returns for each manager are *positively* correlated within a particular style, but that diversification of active returns is especially powerful *across* styles. For example, I assume the active return of the contrarian manager is positively correlated with the active return of the value manager, but negatively correlated with the active return of the growth manager and the momentum manager.[7]

The assumed correlation structure is summarized in Exhibit 7. Note that the average correlation among the active returns in our example is approximately zero ($\overline{\rho} = -0.04$), which is consistent with the values we observe empirically and with the WS assumption that, on average, active returns are uncorrelated.

Exhibit 8 shows optimal portfolio structures for various levels of active risk, given the assumed correlation structure. In addition to the risk-controlled strategy and the concentrated fund, there are four sources of active return and strong complementary characteristics among some sources of active return. In this case, there is very little role for either index strategies or risk-controlled active strategies, except for portfolio structures with a target tracking error of less than 1.5%.

Yet there is a fairly large role for the aggressive active strategies. Because of the added diversification, the optimal portfolios generally allocate more to the contrarian and momentum managers than the more traditional value and growth managers.

Also note the role of the concentrated portfolio. The optimal portfolio with a 2.5% active risk budget

allocates nearly 10% to the concentrated fund with no allocation to indexed or risk-controlled strategies. While the case for the concentrated fund may be overstated here because I assume a constant information ratio across strategies, the optimal portfolio structure in this example is quite different from the Waring and Siegel results.

CONCLUSIONS

The dimensions of active management include the level of active return (alpha), the volatility of active return (tracking error), and the covariance structure of active return. Each dimension is important for understanding optimal portfolio structure.

The science of portfolio construction is intuitive and relatively simple. Investors should seek out diversified (negatively correlated) sources of active return—the more, the better. In the *absence* of diversified sources of active return, investors should include a substantial allocation to risk-controlled active strategies.

While the mathematics may be fairly simple, the practice of portfolio construction—successful implementation—is far more challenging.

ENDNOTES

The author thanks Adrian Cronje, Sam Fraundorf, Rex Macey, and an anonymous referee for valuable comments on previous drafts.

[1]Exhibit 1 replicates portions of the information presented in the appendix of WS. I use throughout the same assumptions about expected active return and expected active risk as WS.

[2]I choose the labels of value and growth to facilitate comparison with the WS example. The results apply more generally, though, we so could label them manager 1 and manager 2.

[3]These active returns are based on a multifactor regression model and are equivalent to what WS label as *pure alpha*.

[4]Given the sample size, any observed correlation with an absolute value in excess of 0.25 is statistically different from zero at the 5% significance level. The fact that nearly half of the observations fall in this range casts serious doubt on the proposition that all of these excess returns are truly independent.

In an additional check, I run a Monte Carlo experiment to investigate how the observed distribution of correlations compares with a distribution based on various samples drawn from active return series that are truly independent. This experiment confirms that the distribution of observed correlations indicates a much higher frequency of significant non-zero (positive or negative) correlations than one would expect if the alpha sources were truly independent.

[5]WS provide two examples where active returns are likely to exhibit non-zero correlations. They suggest that these non-zero correlations are a function of the fact that there are fewer degrees of freedom in these asset classes, noting that active managers in the international arena generally choose to add value via country selection or sector and industry selection while fixed-income managers tend to emphasize either duration or credit quality.

[6]The styles are chosen to represent a logical extension of the example in WS, but the results readily apply to other styles or asset classes. For the contrarian and momentum strategies, I adopt the same assumptions about active return and active risk as for the traditional value and growth strategies.

[7]The fact that these correlations are assumed to exhibit some linkage with management style is completely arbitrary. Because we are working in the realm of active returns (which are by definition uncorrelated with benchmarks), there is no mechanism that guarantees positive correlations among active returns within a style or negative correlations among active returns for managers of different styles.

REFERENCE

Waring, M. Barton, and Laurence B. Siegel. "The Dimensions of Active Management." *The Journal of Portfolio Management*, Spring 2003, pp. 35-51.